

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**B.Sc. DEGREE EXAMINATION – PHYSICS****FIRST SEMESTER – APRIL 2023****MT 1100 – MATHEMATICS FOR PHYSICS**

Date: 08-05-2023

Dept. No.

Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART A**Answer ALL the questions****(10 x 2 =20)**

1. If $y = e^{ax}$, find y_n .
2. Find the polar subtangent and subnormal of the curve $r = a\theta$.
3. Write the expansion for $(1 + x)^n$.
4. Define diagonal matrix.
5. Find the Laplace transform of $t^2 + 2t + 1$.
6. Find $L^{-1}\left(\frac{1}{s-3}\right)$.
7. Write the expansion of $\cos n\theta$.
8. Prove that $\cosh^2 x - \sinh^2 x = 1$.
9. Write down the probability mass function for the Binomial distribution.
10. What is the chance that a leap year selected at random will contain 53 Sundays?

PART B**Answer any FIVE questions****(5 x 8 =40)**

11. Using Leibnitz formula, find the n^{th} derivative of $x^2 \log x$.
12. Find the angle of intersection of the cardioids $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.
13. Find the sum to infinity of the series $1 + \frac{3}{2!} + \frac{5}{3!} + \frac{7}{4!} + \dots \infty$.
14. If $A = \begin{pmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$, then show that $(AB)^T = B^T A^T$.
15. Find $L^{-1}\left(\frac{1}{s(s+1)(s+9)}\right)$.
16. Write down the expansion of $\sin^6 \theta$ in series of cosines of multiples of θ .
17. If $\sin(A + iB) = x + iy$, prove that $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$ and $\frac{x^2}{\cosh^2 A} + \frac{y^2}{\sinh^2 A} = 1$.
18. Two unbiased dice are thrown. Find the probability that (i) both the dice show the same number (ii) the first die shows 6 (iii) the total of the numbers on the dice is 8 (iv) the total of the numbers on the dice is greater than 8.

PART C**Answer any TWO questions****(2 x 20 =40)**

19. (a) Find the length of the subtangent, subnormal, tangent and normal at (a, a) on the cissoid $y^2 = \frac{x^3}{2a-x}$.
- (b) Find the maximum and minimum values of the function $f(x) = 2x^3 - 3x^2 - 36x + 10$.

(10+10)

20. (a) Find the characteristic roots and associated characteristic vectors for the matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}.$$

(b) Verify Cayley- Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$. (12+8)

21. (a) Find (i) $L(e^{-5t}\cos^2 t)$ (ii) $L(te^{-t}\sin t)$.

(b) If $\tan(x + iy) = u + iv$, prove that $\frac{u}{v} = \frac{\sin 2x}{\sinh 2y}$. (10+10)

22. (a) Find $\lim_{\theta \rightarrow 0} \left(\frac{n\sin\theta - \sin n\theta}{\theta(\cos\theta - \sin n\theta)} \right)$.

(b) Calculate the mean and standard deviation for the following table giving the age distribution of 542 members.

Age in years	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of Members	3	61	132	153	140	51	2

(8+12)

\$\$\$\$\$\$